Unification and fermion mass structure.

Graham G. Ross *and Mario Serna †

Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP

June 11, 2008

Abstract

Grand Unified Theories predict relationships between the GUT-scale quark and lepton masses. Using new data in the context of the MSSM, we update the values and uncertainties of the masses and mixing angles for the three generations at the GUT scale. We also update fits to hierarchical patterns in the GUT-scale Yukawa matrices. The new data shows not all the classic GUT-scale mass relationships remain in quantitative agreement at small to moderate $\tan\beta$. However, at large $\tan\beta$, these discrepancies can be eliminated by finite, $\tan\beta$-enhanced, radiative, threshold corrections if the gluino mass has the opposite sign to the wino mass.

Explaining the origin of fermion masses and mixings remains one of the most important goals in our attempts to go beyond the Standard Model. In this, one very promising possibility is that there is an underlying stage of unification relating the couplings responsible for the fermion masses. However we are hindered by the fact that the measured masses and mixings do not directly give the structure of the underlying Lagrangian both because the data is insufficient unambiguously to reconstruct the full fermion mass matrices and because radiative corrections can obscure the underlying structure. In this letter we will address both these points in the context of the MSSM.

We first present an analysis of the measured mass and mixing angles continued to the GUT scale. The analysis updates previous work, using the precise measurements of fermion masses and mixing angles from the b-factories and the updated top-quark mass from CDF and D0. The resulting data at the GUT scale allows us to look for underlying patterns which may suggest a unified origin. We also explore the sensitivity of these patterns to $\tan\beta$-enhanced, radiative threshold corrections.

We next proceed to extract the underlying Yukawa coupling matrices for the quarks and leptons. There are two difficulties in this. The first is that the data cannot, without some assumptions, determine all elements of these matrices. The second is that the Yukawa coupling matrices are basis dependent. We choose to work in a basis in which the mass matrices are hierarchical in structure with the off-diagonal elements small relative to the appropriate combinations of on-diagonal matrix elements. This is the basis we think is most likely to display the structure of the underlying theory, for example that of a spontaneously broken family symmetry in which the hierarchical structure is ordered by the (small) order parameter breaking the symmetry. With this structure to leading order the observed masses and mixing angles determine the mass matrix elements on and above the diagonal, and our analysis determines these entries, again allowing for significant $\tan\beta$ enhanced radiative corrections. The resulting form of the mass matrices provides the "data" for developing models of fermion masses such as those based on a broken family symmetry.

The data set used is summarized in Table 1. Since the fit of reference [4] (RRRV) to the Yukawa texture was done, the measurement of the Standard-Model parameters has improved considerably. We highlight a few of the changes in the data since 2000: The top-quark mass has gone from $M_t = 174.3 \pm 5$ GeV to $M_t = 170.9 \pm 1.9$ GeV. In 2000 the Particle Data Book reported $m_b(m_b) = 4.2 \pm 0.2$ GeV [5] which has improved to $m_b(m_b) = 4.2 \pm 0.07$ GeV today. In addition each higher order QCD correction pushes down the value of $m_b(M_Z)$ at the scale of the Z bosons mass. In 1998 $m_b(M_Z) = 3.0 \pm 0.2$ GeV [6] and today it is $m_b(M_Z) = 2.87 \pm 0.06$ GeV [7]. The most significant shift in the data relevant to the RRRV fit is a downward revision to the strange-quark mass at the scale $\mu_L = 2$ GeV from $m_s(\mu_L) \approx 120 \pm 50$ MeV [5] to

*g.ross@physics.ox.ac.uk

†serna@physics.ox.ac.uk

Low-Energy Parameter	Value(Uncertainty in last digit(s))	Notes and Reference
$m_u(\mu_L)/m_d(\mu_L)$	0.45(15)	PDB Estimation [1]
$m_s(\mu_L)/m_d(\mu_L)$	19.5(1.5)	PDB Estimation [1]
$m_u(\mu_L)+m_d(\mu_L)$	[8.8(3.0), 7.6(1.6)] MeV	PDB, Quark Masses, pg 15 [1]. (Non-lattice, Lattice)
$Q=\sqrt{\frac{m_s^2-(m_d+m_u)^2/4}{m_d^2-m_u^2}}$	22.8(4)	Martemyanov and Sopov [2]
$m_s(\mu_L)$	[103(20) , 95(20)] MeV	PDB, Quark Masses, pg 15 [1]. [Non-lattice, lattice]
$m_u(\mu_L)$	3(1) MeV	PDB, Quark Masses, pg 15 [1]. Non-lattice.
$m_d(\mu_L)$	6.0(1.5) MeV	PDB, Quark Masses, pg 15 [1]. Non-lattice.
$m_c(m_c)$	1.24(09) GeV	PDB, Quark Masses, pg 16 [1]. Non-lattice.
$m_b(m_b)$	4.20(07) GeV	PDB, Quark Masses, pg 16,19 [1]. Non-lattice.
M_t	170.9 (1.9) GeV	CDF & D0 [3] Pole Mass
(M_e, M_μ, M_τ)	(0.511(15), 105.6(3.1), 1777(53)) MeV	3% uncertainty from neglecting Y^e thresholds.
A Wolfenstein parameter	0.818(17)	PDB Ch 11 Eq. 11.25 [1]
$\overline{\rho}$ Wolfenstein parameter	0.221(64)	PDB Ch 11 Eq. 11.25 [1]
λ Wolfenstein parameter	0.2272(10)	PDB Ch 11 Eq. 11.25 [1]
$\overline{\eta}$ Wolfenstein parameter	0.340(45)	PDB Ch 11 Eq. 11.25 [1]
$\lvert V_{CKM}\rvert$	$\begin{pmatrix} 0.97383(24) & 0.2272(10) & 0.00396(09) \\ 0.2271(10) & 0.97296(24) & 0.04221(80) \\ 0.00814(64) & 0.04161(78) & 0.999100(34) \end{pmatrix}$	PDB Ch 11 Eq. 11.26 [1]
$\sin 2\beta$ from CKM	0.687(32)	PDB Ch 11 Eq. 11.19 [1]
Jarlskog Invariant	$3.08(18)\times 10^{-5}$	PDB Ch 11 Eq. 11.26 [1]
$v_{Higgs}(M_Z)$	246.221(20) GeV	Uncertainty expanded. [1]
($\alpha_{EM}^{-1}(M_Z)$, $\alpha_s(M_Z)$, $\sin^2\theta_W(M_Z)$)	(127.904(19), 0.1216(17), 0.23122(15))	PDB Sec 10.6 [1]

Table 1: Low-energy observables. Masses in lower-case m are $\overline{MS}$ running masses. Capital M indicates pole mass. The light quark's (u,d,s) mass are specified at a scale $\mu_L = 2$ GeV. V_{CKM} are the Standard Model's best fit values.

today's value $m_s(\mu_L) = 103 \pm 20$ MeV. We also know the CKM unitarity triangle parameters better today than six years ago. For example, in 2000 the Particle Data book reported $\sin 2\beta = 0.79 \pm 0.4$ [5] which is improved to $\sin 2\beta = 0.69 \pm 0.032$ in 2006 [1]. The $\sin 2\beta$ value is about $1.2\,\sigma$ off from a global fit to all the CKM data [8], our fits generally lock onto the global-fit data and exhibit a $1\,\sigma$ tension for $\sin 2\beta$. Together, the improved CKM matrix observations add stronger constraints to the textures compared to data from several years ago.

We first consider the determination of the fundamental mass parameters at the GUT scale in order simply to compare to GUT predictions. The starting point for the light-quark masses at low scale is given by the χ^2 fit to the data of Table 1

$$m_u(\mu_L) = 2.7 \pm 0.5 \text{ MeV} \quad m_d(\mu_L) = 5.3 \pm 0.5 \text{ MeV} \quad m_s(\mu_L) = 103 \pm 12 \text{ MeV}. \tag{1}$$

Using these as input we determine the values of the mass parameters at the GUT scale for various choices of $\tan\beta$ but not including possible $\tan\beta$ enhanced threshold corrections. We do this using numerical solutions to the RG equations. The one-loop and two-loop RG equations for the gauge couplings and the Yukawa couplings in the Standard Model and in the MSSM that we use in this study come from a number of sources [6] [9][10] [11]. The results are given in the first five columns of Table 2. These can readily be compared to expectations in various Grand Unified models. The classic prediction of $SU(5)$ with third generation down-

quark and charged-lepton masses given by the coupling $B\,\overline{5}_f.10_f.5_H$ [1] is $m_b(M_X)/m_\tau(M_X) = 1$ [12]. This ratio is given in Table 2 where it may be seen that the value agrees at a special low $\tan\beta$ value but for large $\tan\beta$ it is some 25% smaller than the GUT prediction[2]. A similar relation between the strange quark and the muon is untenable and to describe the masses consistently in $SU(5)$ Georgi and Jarlskog [14] proposed that the second generation masses should come instead from the coupling $C\,\overline{5}_f.10_f.45_H$ leading instead to the relation $3m_s(M_X)/m_\mu(M_X) = 1$. As may be seen from Table 2 in all cases this ratio is approximately 0.69(8). The prediction of Georgi and Jarlskog for the lightest generation masses follows from the relation $Det(M^d)/Det(M^l) = 1$. This results from the form of their mass matrix which is given by[3]

$$M^d = \begin{pmatrix} 0 & A' & \\ A & C & \\ & & B \end{pmatrix},\; M^l = \begin{pmatrix} 0 & A' & \\ A & -3C & \\ & & B \end{pmatrix} \tag{2}$$

in which there is a $(1,1)$ texture zero[4] and the determinant is given by the product of the $(3,3)$, $(1,2)$ and $(2,1)$ elements. If the $(1,2)$ and $(2,1)$ elements are also given by $\overline{5}_f.10_f.5_H$ couplings they will be the same in the down-quark and charged-lepton mass matrices giving rise to the equality of the determinants. The form of eq(2) may be arranged by imposing additional continuous or discrete symmetries. One may see from Table 2 that the actual value of the ratio of the determinants is quite far from unity disagreeing with the Georgi Jarlskog relation.

In summary the latest data on fermion masses, while qualitatively in agreement with the simple GUT relations, has significant quantitative discrepancies. However the analysis has not, so far, included the SUSY threshold corrections which substantially affect the GUT mass relations at large $\tan\beta$ [15]. A catalog of the full SUSY threshold corrections is given in [16]. The particular finite SUSY thresholds discussed in this letter do not decouple as the super partners become massive. We follow the approximation described in Blazek, Raby, and Pokorski (BRP) for threshold corrections to the CKM elements and down-like mass eigenstates [17]. The finite threshold corrections to Y^e and Y^u and are generally about 3% or smaller

$$\delta Y^u,\; \delta Y^d \lesssim 0.03 \tag{3}$$

and will be neglected in our study. The logarithmic threshold corrections are approximated by using the Standard-Model RG equations from M_Z to an effective SUSY scale M_S.

The finite, $\tan\beta$-enhanced Y^d SUSY threshold corrections are dominated by the a sbottom-gluino loop, a stop-higgsino loop, and a stop-chargino loop. Integrating out the SUSY particles at a scale M_S leaves the matching condition at that scale for the Standard-Model Yukawa couplings:

$$\delta m_{sch}\, Y^{u\,SM} = \sin\beta\; Y^u \tag{4}$$

$$\delta m_{sch}\, Y^{d\,SM} = \cos\beta\; U_L^{d\dagger}\left(1 + \Gamma^d + V_{CKM}^\dagger\,\Gamma^u\, V_{CKM}\right) Y^d_{\rm diag}\, U_R^d \tag{5}$$

$$Y^{e\,SM} = \cos\beta\; Y^e. \tag{6}$$

All the parameters on the right-hand side take on their MSSM values in the $\overline{DR}$ scheme. The factor δm_{sch} converts the quark running masses from $\overline{MS}$ to $\overline{DR}$ scheme. The β corresponds to the ratio of the two Higgs VEVs $v_u/v_d = \tan\beta$. The U matrices decompose the MSSM Yukawa couplings at the scale M_S: $Y^u = U_L^{u\dagger} Y^u_{\rm diag} U^u_R$ and $Y^d = U_L^{d\dagger} Y^d_{\rm diag} U^d_R$. The matrices $Y^u_{\rm diag}$ and $Y^d_{\rm diag}$ are diagonal and correspond to the mass eigenstates divided by the appropriate VEV at the scale M_S. The CKM matrix is given by $V_{CKM} = U_L^u U_L^{d\dagger}$. The left-hand side involves the Standard-Model Yukawa couplings. The matrices Γ^u and Γ^d encode the SUSY threshold corrections.

If the squarks are diagonalized in flavor space by the same rotations that diagonalize the quarks, the matrices Γ^u and Γ^d are diagonal: $\Gamma^d = {\rm diag}(\gamma_d, \gamma_d, \gamma_b)$, $\Gamma^u = {\rm diag}(\gamma_u, \gamma_u, \gamma_t)$. In general the squarks are

[1] $\overline{5}_f$, 10_f refer to the $SU(5)$ representations making up a family of quarks and leptons while 5_H is a five dimensional representation of Higgs scalars.

[2] We'd like to thank Ilja Dorsner for pointing out that the $\tan\beta$ dependence of $m_b/m_\tau(M_X)$ is more flat than in previous studies (e.g. ref. [13]). This change is mostly due to the higher effective SUSY scale M_S, the higher value of $\alpha_s(M_Z)$ found in global standard model fits, and smaller top-quark mass M_t.

[3] The remaining mass matrix elements may be non-zero provided they do not contribute significantly to the deteminant

[4] Below we discuss an independent reason for having a $(1,1)$ texture zero.

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	Corresponding GUT-Scale Parameters with Propagated Uncertainty					
$y^t(M_X)$	6^{+1}_{-5}	0.48(2)	0.49(2)	0.51(3)	0.51(2)	0.51(2)
$y^b(M_X)$	$0.0113^{+0.0002}_{-0.01}$	0.051(2)	0.23(1)	0.37(2)	0.34(3)	0.34(3)
$y^\tau(M_X)$	0.0114(3)	0.070(3)	0.32(2)	0.51(4)	0.34(2)	0.34(2)
$(m_u/m_c)(M_X)$	0.0027(6)	0.0027(6)	0.0027(6)	0.0027(6)	0.0026(6)	0.0026(6)
$(m_d/m_s)(M_X)$	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)
$(m_e/m_\mu)(M_X)$	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)
$(m_c/m_t)(M_X)$	$0.0009^{+0.001}_{-0.00006}$	0.0025(2)	0.0024(2)	0.0023(2)	0.0023(2)	0.0023(2)
$(m_s/m_b)(M_X)$	0.014(4)	0.019(2)	0.017(2)	0.016(2)	0.018(2)	0.010(2)
$(m_\mu/m_\tau)(M_X)$	0.059(2)	0.059(2)	0.054(2)	0.050(2)	0.054(2)	0.054(2)
$A(M_X)$	$0.56^{+0.34}_{-0.01}$	0.77(2)	0.75(2)	0.72(2)	0.73(3)	0.46(3)
$\lambda(M_X)$	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)
$\bar\rho(M_X)$	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)
$\bar\eta(M_X)$	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)
$J(M_X)\times 10^{-5}$	$1.4^{+2.2}_{-0.2}$	2.6(4)	2.5(4)	2.3(4)	2.3(4)	1.0(2)
Parameters	Comparison with GUT Mass Ratios					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

Table 2: The mass parameters continued to the GUT-scale M_X for various values of $\tan\beta$ and threshold corrections $\gamma_{t,b,d}$. These are calculated with the 2-loop gauge coupling and 2-loop Yukawa coupling RG equations assuming an effective SUSY scale $M_S = 500$ GeV.

not diagonalized by the same rotations as the quarks but provided the relative mixing angles are reasonably small the corrections to flavour conserving masses, which are our primary concern here, will be second order in these mixing angles. We will assume Γ^u and Γ^d are diagonal in what follows.

Approximations for Γ^u and Γ^d based on the mass insertion approximation are found in [18][19][20]:

$$\begin{aligned}
\gamma_t &\approx y_t^2\,\mu\,A^t\,\frac{\tan\beta}{16\pi^2} I_3(m^2_{\tilde t_1}, m^2_{\tilde t_2}, \mu^2) \sim y_t^2\,\frac{\tan\beta}{32\pi^2}\frac{\mu\,A^t}{m^2_{\tilde t}} &(7)\\
\gamma_u &\approx -g_2^2\,M_2\,\mu\,\frac{\tan\beta}{16\pi^2} I_3(m^2_{\chi_1}, m^2_{\chi_2}, m^2_{\tilde u}) \sim 0 &(8)\\
\gamma_b &\approx \frac{8}{3}\,g_3^2\,\frac{\tan\beta}{16\pi^2}\,M_3\,\mu\,I_3(m^2_{\tilde b_1}, m^2_{\tilde b_2}, {M_3}^2) \sim \frac{4}{3}\,g_3^2\,\frac{\tan\beta}{16\pi^2}\,\frac{\mu\,M_3}{m^2_{\tilde b}} &(9)\\
\gamma_d &\approx \frac{8}{3}g_3^2\frac{\tan\beta}{16\pi^2}M_3\,\mu\,I_3(m^2_{\tilde d_1}, m^2_{\tilde d_2}, {M_3}^2) \sim \frac{4}{3}\,g_3^2\,\frac{\tan\beta}{16\pi^2}\,\frac{\mu\,M_3}{m^2_{\tilde d}} &(10)
\end{aligned}$$

where I_3 is given by

$$I_3(a^2, b^2, c^2) = \frac{a^2 b^2 \log\frac{a^2}{b^2} + b^2 c^2 \log\frac{b^2}{c^2} + c^2 a^2 \log\frac{c^2}{a^2}}{(a^2-b^2)(b^2-c^2)(a^2-c^2)}. \qquad (11)$$

In these expressions $\tilde q$ refers to superpartner of q. χ^j indicate chargino mass eigenstates. μ is the coefficient to the $H^u\,H^d$ interaction in the superpotential. M_1, M_2, M_3 are the gaugino soft breaking terms. A^t refers to the soft top-quark trilinear coupling. The mass insertion approximation breaks down if there is large mixing between the mass eigenstates of the stop or the sbottom. The right-most expressions in eqs(7,9,10) assume the relevant squark mass eigenstates are nearly degenerate and heavier than M_3 and μ. These expressions (

eqs 7 - 10) provide an approximate mapping from a supersymmetric spectra to the γ_i parameters through which we parameterize the threshold corrections; however, with the exception of Column A of Table 4, we do not specify a SUSY spectra but directly parameterize the thresholds corrections through γ_i.

The separation between γ_b and γ_d is set by the lack of degeneracy of the down-like squarks. If the squark masses for the first two generations are not degenerate, then there will be a corresponding separation between the (1,1) and (2,2) entries of Γ^d and Γ^u. If the sparticle spectra is designed to have a large A^t and a light stop, γ_t can be enhanced and dominate over γ_b. Because the charm Yukawa coupling is so small, the scharm-higgsino loop is negligible, and γ_u follows from a chargino squark loop and is also generally small with values around 0.02 because of the smaller g_2 coupling. In our work, we approximate $\Gamma^u_{22} \sim \Gamma^u_{11} \sim 0$. The only substantial correction to the first and second generations is given by γ_d [15].

As described in BRP, the threshold corrections leave $|V_{us}|$ and $|V_{ub}/V_{cb}|$ unchanged to a good approximation. Threshold corrections in Γ^u do affect the V_{ub} and V_{cb} at the scale M_S giving

$$\frac{V_{ub}^{SM} - V_{ub}^{MSSM}}{V_{ub}^{MSSM}} \backsimeq \frac{V_{cb}^{SM} - V_{cb}^{MSSM}}{V_{cb}^{MSSM}} \backsimeq -\left(\gamma_t - \gamma_u\right). \tag{12}$$

The threshold corrections for the down-quark masses are given approximately by

$$\begin{aligned} m_d &\backsimeq m_d^0 \left(1 + \gamma_d + \gamma_u\right)^{-1} \\ m_s &\backsimeq m_s^0 \left(1 + \gamma_d + \gamma_u\right)^{-1} \\ m_b &\backsimeq m_b^0 \left(1 + \gamma_b + \gamma_t\right)^{-1} \end{aligned}$$

where the superscript 0 denotes the mass without threshold corrections. Not shown are the nonlinear effects which arise through the RG equations when the bottom Yukawa coupling is changed by threshold effects. These are properly included in our final results obtained by numerically solving the RG equations.

Due to our assumption that the squark masses for the first two generations are degenerate, the combination of the GUT relations given by $\left(\det M^l / \det M^d\right) \left(3\, m_s/m_\mu\right)^2 \left(m_b/m_\tau\right) = 1$ is unaffected up to nonlinear effects. Thus we cannot simultaneously fit all three GUT relations through the threshold corrections. A best fit requires the threshold effects given by

$$\gamma_b + \gamma_t \approx -0.22 \pm 0.02 \tag{13}$$

$$\gamma_d + \gamma_u \approx -0.21 \pm 0.02. \tag{14}$$

giving the results shown in the penultimate column of Table 2, just consistent with the GUT predictions. The question is whether these threshold effects are of a reasonable magnitude and, if so, what are the implications for the SUSY spectra which determine the γ_i? From eqs(9,10), at $\tan\beta = 38$ we have

$$\frac{\mu\, M_3}{m_{\tilde{b}}^2} \sim -0.5, \qquad \frac{m_{\tilde{b}}^2}{m_{\tilde{d}}^2} \sim 1.0$$

The current observation of the muon's $(g-2)_\mu$ is $3.4\,\sigma$ [21] away from the Standard-Model prediction. If SUSY is to explain the observed deviation, one needs $\tan\beta > 8$ [22] and $\mu M_2 > 0$ [23]. With this sign we must have μM_3 negative and the $\widetilde{d}$, $\widetilde{s}$ squarks only lightly split from the $\widetilde{b}$ squarks. M_3 negative is characteristic of anomaly mediated SUSY breaking[24] and is discussed in [25][26][20][27]. Although we have deduced $M_3 < 0$ from the approximate eqs(9,10), the the correlation persists in the near exact expression found in eq(23) of ref [17]. Adjusting to different squark splitting can occur in various schemes[28]. However the squark splitting can readily be adjusted without spoiling the fit because, up to nonlinear effects, the solution only requires the constraints implied by eq(13), so we may make $\gamma_b > \gamma_d$ and hence make $m_{\tilde{b}}^2 < m_{\tilde{d}}^2$ by allowing for a small positive value for γ_t. In this case A^t must be positive.

It is of interest also to consider the threshold effects in the case that μM_3 is positive. This is illustrated in the last column of Table 2 in which we have reversed the sign of γ_d, consistent with positive μM_3 , and chosen $\gamma_b \simeq \gamma_d$ as is expected for similar down squark masses. The value of γ_t is chosen to keep the equality between m_b and m_τ. One may see that the other GUT relations are not satisfied, being driven further away by the threshold corrections. Reducing the magnitude of γ_b and γ_d reduces the discrepancy somewhat but still limited by the deviation found in the no-threshold case (the fourth column of Table 2).

Parameter	2001 RRRV	Fit A0	Fit B0	Fit A1	Fit B1	Fit A2	Fit B2
$\tan\beta$	Small	1.3	1.3	38	38	38	38
a'	$\mathcal{O}(1)$	0	0	0	0	-2.0	-2.0
ϵ_u	0.05	0.030(1)	0.030(1)	0.0491(16)	0.0491(15)	0.0493(16)	0.0493(14)
ϵ_d	0.15(1)	0.117(4)	0.117(4)	0.134(7)	0.134(7)	0.132(7)	0.132(7)
$\lvert b'\rvert$	1.0	1.75(20)	1.75(21)	1.05(12)	1.05(13)	1.04(12)	1.04(13)
$\arg(b')$	90^o	$+93(16)^o$	$-93(13)^o$	$+91(16)^o$	$-91(13)^o$	$+93(16)^o$	$-93(13)^o$
a	1.31(14)	2.05(14)	2.05(14)	2.16(23)	2.16(24)	1.92(21)	1.92(22)
b	1.50(10)	1.92(14)	1.92(15)	1.66(13)	1.66(13)	1.70(13)	1.70(13)
$\lvert c\rvert$	0.40(2)	0.85(13)	2.30(20)	0.78(15)	2.12(36)	0.83(17)	2.19(38)
$\arg(c)$	$-24(3)^o$	$-39(18)^o$	$-61(14)^o$	$-43(14)^o$	$-59(13)^o$	$-37(25)^o$	$-60(13)^o$

Table 3: Results of a χ^2 fit of eqs(15,16) to to the data in Table 2 in the absence of threshold corrections. We set a' as indicated and set $c' = d' = d = 0$ and $f = f' = 1$ at fixed values.

At $\tan\beta$ near 50 the non-linear effects are large and $b-\tau$ unification requires $\gamma_b + \gamma_t \sim -0.1$ to -0.15. In this case it is possible to have $t-b-\tau$ unification of the Yukawa couplings. For $\mu > 0, M_3 > 0$, the "Just-so" Split-Higgs solution of references [29, 30, 31, 32] can achieve this while satisfying both $b \to s\,\gamma$ and $(g-2)_\mu$ constraints but only with large γ_b and γ_t and a large cancellation in $\gamma_b + \gamma_t$. In this case, as in the example given above, the threshold corrections drive the masses further from the mass relations for the first and second generations because $\mu\, M_3 > 0$. It *is* possble to have $t-b-\tau$ unification with $\mu\, M_3 < 0$, satisfying the $b \to s\,\gamma$ and $(g-2)_\mu$ constraints in which the GUT predictions for the first and second generation of quarks is acceptable. Examples include Non-Universal Gaugino Mediation [33] and AMSB; both have some very heavy sparticle masses ($\gtrsim 4$ TeV) [20]. Minimal AMSB with a light sparticle spectra($\lesssim 1$ TeV), while satisfying $(g-2)_\mu$ and $b \to s\,\gamma$ constraints, requires $\tan\beta$ less than about 30 [23].

We turn now to the second part of our study in which we update previous fits to the Yukawa matrices responsible for quark and lepton masses. As discussed above we choose to work in a basis in which the mass matrices are hierarchical with the off-diagonal elements small relative to the appropriate combinations of on-diagonal matrix elements. This is the basis we think is most likely to display the structure of the underlying theory, for example that of a spontaneously broken family symmetry, in which the hierarchical structure is ordered by the (small) order parameter breaking the symmetry. With this structure to leading order in the ratio of light to heavy quarks the observed masses and mixing angles determine the mass matrix elements on and above the diagonal provided the elements below the diagonal are not anomalously large. This is the case for matrices that are nearly symmetrical or for nearly Hermitian as is the case in models based on an $SO(10)$ GUT.

For convenience we fit to symmetric Yukawa coupling matrices but, as stressed above, this is not a critical assumption as the data is insensitive to the off-diagonal elements below the diagonal and the quality of the fit is not changed if, for example, we use Hermitian forms. We parameterize a set of general, symmetric Yukawa matrices as:

$$Y^u(M_X) = y^u_{33} \begin{pmatrix} d'\,\epsilon_u^4 & b'\,\epsilon_u^3 & c'\,\epsilon_u^3 \\ b'\,\epsilon_u^3 & f'\,\epsilon_u^2 & a'\,\epsilon_u^2 \\ c'\,\epsilon_u^3 & a'\,\epsilon_u^2 & 1 \end{pmatrix}, \tag{15}$$

$$Y^d(M_X) = y^d_{33} \begin{pmatrix} d\,\epsilon_d^4 & b\,\epsilon_d^3 & c\,\epsilon_d^3 \\ b\,\epsilon_d^3 & f\,\epsilon_d^2 & a\,\epsilon_d^2 \\ c\,\epsilon_d^3 & a\,\epsilon_d^2 & 1 \end{pmatrix}. \tag{16}$$

Although not shown, we always choose lepton Yukawa couplings at M_X consistent with the low-energy lepton masses. Notice that the f coefficient and ϵ_d are redundant (likewise in Y^u). We include f to be able to discuss the phase of the (2,2) term. We write all the entries in terms of ϵ so that our coefficients will be $\mathcal{O}(1)$. We will always select our best ϵ parameters such that $|f| = 1$.

RRRV noted that all solutions, to leading order in the small expansion parameters, only depend on two

Parameter	A	B	C	B2	C2
$\tan\beta$	30	38	38	38	38
γ_b	0.20	-0.22	$+0.22$	-0.22	$+0.22$
γ_t	-0.03	0	-0.44	0	-0.44
γ_d	0.20	-0.21	$+0.21$	-0.21	$+0.21$
a'	0	0	0	-2	-2
ϵ_u	0.0495(17)	0.0483(16)	0.0483(18)	0.0485(17)	0.0485(18)
ϵ_d	0.131(7)	0.128(7)	0.102(9)	0.127(7)	0.101(9)
$\lvert b'\rvert$	1.04(12)	1.07(12)	1.07(11)	1.05(12)	1.06(10)
$\arg(b')$	$90(12)^o$	$91(12)^o$	$93(12)^o$	$95(12)^o$	$95(12)^o$
a	2.17(24)	2.27(26)	2.30(42)	2.03(24)	1.89(35)
b	1.69(13)	1.73(13)	2.21(18)	1.74(10)	2.26(20)
$\lvert c\rvert$	0.80(16)	0.86(17)	1.09(33)	0.81(17)	1.10(35)
$\arg(c)$	$-41(18)^o$	$-42(19)^o$	$-41(14)^o$	$-53(10)^o$	$-41(12)^o$
Y^u_{33}	0.48(2)	0.51(2)	0.51(2)	0.51(2)	0.51(2)
Y^d_{33}	0.15(1)	0.34(3)	0.34(3)	0.34(3)	0.34(3)
Y^e_{33}	0.23(1)	0.34(2)	0.34(2)	0.34(2)	0.34(2)
$(m_b/m_\tau)(M_X)$	0.67(4)	1.00(4)	1.00(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	0.60(3)	0.9(1)	0.6(1)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.71(7)	1.04(8)	0.68(6)	1.04(8)	0.68(6)
$\left\lvert\frac{\det Y^d(M_X)}{\det Y^e(M_X)}\right\rvert$	0.3(1)	0.92(14)	0.4(1)	0.92(14)	0.4(1)

Table 4: A χ^2 fit of eqs(15,16) including the SUSY threshold effects parameterized by the specified γ_i.

phases ϕ_1 and ϕ_2 given by

$$\phi_1 = (\phi'_b - \phi'_f) - (\phi_b - \phi_f) \tag{17}$$
$$\phi_2 = (\phi_c - \phi_a) - (\phi_b - \phi_f). \tag{18}$$

where ϕ_x is the phase of parameter x. For this reason it is sufficient to consider only b' and c as complex with all other parameters real.

As mentioned above the data favours a texture zero in the $(1,1)$ position. With a symmetric form for the mass matrix for the first two families, this leads to the phenomenologically successful Gatto Sartori Tonin [34] relation

$$V_{us}(M_X) \approx \left|b\epsilon_d - |b'|e^{i\,\phi_{b'}}\epsilon_u\right| \approx \left|\sqrt{(\frac{m_d}{m_s})_0} - \sqrt{(\frac{m_u}{m_c})_0}e^{i\,\phi_1}\right|. \tag{19}$$

This relation gives an excellent fit to V_{us} with $\phi_1 \approx \pm 90^o$, and to preserve it we take d, d' to be zero in our fits. As discussed above, in $SU(5)$ this texture zero leads to the GUT relation $Det(M^d)/Det(M^l) = 1$ which, with threshold corrections, is in good agreement with experiment. In the case that c is small it was shown in RRRV that ϕ_1 is to a good approximation the CP violating phase δ in the Wolfenstein parameterization. A non-zero c is necessary to avoid the relation $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ and with the improvement in the data, it is now necessary to have c larger than was found in RRRV [5]. As a result the contribution to CP violation coming from ϕ_2 is at least 30%. The sign ambiguity in ϕ_1 gives rise to an ambiguity in c with the positive sign corresponding to the larger value of c seen in Tables 3 and 4.

Table 3 shows results from a χ^2 fit of eqs(15,16) to to the data in Table 2 in the absence of threshold corrections. The error, indicated by the term in brackets, represent the widest axis of the 1σ error ellipse in parameter space. The fits labeled 'A' have phases such that we have the smaller magnitude solution of $|c|$, and fits labeled 'B' have phases such that we have the larger magnitude solution of $|c|$. As discussed above, it is not possible unambiguously to determine the relative contributions of the off-diagonal elements of the up and down Yukawa matrices to the mixing angles. In the fit A2 and B2 we illustrate the uncertainty

[5] As shown in ref. [35], it is possible, in a basis with large off-diagonal entries, to have an Hermitian pattern with the (1,1) and (1,3) zero provided one carefully orchestrates cancelations among Y^u and Y^d parameters. We find this approach requires a strange-quark mass near its upper limit.

associated with this ambiguity, allowing for $O(1)$ coefficients a'. In all the examples in Table 3, the mass ratios, and Wolfenstein parameters are essentially the same as in Table 2.

The effects of the large $\tan\beta$ threshold corrections are shown in Table 4. The threshold corrections depend on the details of the SUSY spectrum, and we have displayed the effects corresponding to a variety of choices for this spectrum. Column A corresponds to a "standard" SUGRA fit - the benchmark Snowmass Points and Slopes (SPS) spectra 1b of ref([36]). Because the spectra SPS 1b has large stop and sbottom squark mixing angles, the approximations given in eqns(7-10) break down, and the value for the correction γ_i in Column A need to be calculated with the more complete expressions in BRP [17]. In the column A fit and the next two fits in columns B and C, we set a' and c' to zero. Column B corresponds to the fit given in the penultimate column of Table 2 which agrees very well with the simple GUT predictions. It is characterized by the "anomaly-like" spectrum with M_3 negative. Column C examines the M_3 positive case while maintaining the GUT prediction for the third generation $m_b = m_\tau$. It corresponds to the "Just-so" Split-Higgs solution. In the fits A, B and C the value of the parameter a is significantly larger than that found in RRRV. This causes problems for models based on non-Abelian family symmetries, and it is of interest to try to reduce a by allowing a', b' and c' to vary while remaining $\mathcal{O}(1)$ parameters. Doing this for the fits B and C leads to the fits B2 and C2 given in Table 4 where it may be seen that the extent to which a can be reduced is quite limited. Adjusting to this is a challenge for the broken family-symmetry models.

Although we have included the finite corrections to match the MSSM theory onto the standard model at an effective SUSY scale $M_S = 500$ GeV, we have not included finite corrections from matching onto a specific GUT model. Precise threshold corrections cannot be rigorously calculated without a specific GUT model. Here we only estimate the order of magnitude of corrections to the mass relations in Table 2 from matching the MSSM values onto a GUT model at the GUT scale. The $\tan\beta$ enhanced corrections in eq(7-10) arise from soft SUSY breaking interactions and are suppressed by factors of M_{SUSY}/M_{GUT} in the high-scale matching. Allowing for $\mathcal{O}(1)$ splitting of the mass ratios of the heavy states, one obtains corrections to y^b/y^τ (likewise for the lighter generations) of $\mathcal{O}(\frac{g^2}{(4\pi)^2})$ from the X and Y gauge bosons and $\mathcal{O}(\frac{y_b^2}{(4\pi)^2})$ from colored Higgs states. Because we have a different Higgs representations for different generations, these threshold correction will be different for correcting the $3m_s/m_\mu$ relation than the m_b/m_τ relation. These factors can be enhanced in the case there are multiple Higgs representation. For an $SU(5)$ SUSY GUT these corrections are of the order of $2\,\%$. Plank scale suppressed operators can also induce corrections to both the unification scale [37] and may have significant effects on the masses of the lighter generations [38]. In the case that the Yukawa texture is given by a broken family symmetry in terms of an expansion parameter ϵ, one expects model dependent corrections of order ϵ which may be significant.

In summary, in the light of the significant improvement in the measurement of fermion mass parameters, we have analyzed the possibility that the fermion mass structure results from an underlying supersymmetric GUT at a very high-scale mirroring the unification found for the gauge couplings. Use of the RG equations to continue the mass parameters to the GUT scale shows that, although qualitatively in agreement with the GUT predictions coming from simple Higgs structures, there is a small quantitative discrepancy. We have shown that these discrepancies may be eliminated by finite radiative threshold corrections involving the supersymmetric partners of the Standard-Model states. The required magnitude of these corrections is what is expected at large $\tan\beta$, and the form needed corresponds to a supersymmetric spectrum in which the gluino mass is negative with the opposite sign to the Wino mass. We have also performed a fit to the recent data to extract the underlying Yukawa coupling matrices for the quarks and leptons. This is done in the basis in which the mass matrices are hierarchical in structure with the off-diagonal elements small relative to the appropriate combinations of on-diagonal matrix elements, the basis most likely to be relevant if the fermion mass structure is due to a spontaneously broken family symmetry. We have explored the effect of SUSY threshold corrections for a variety of SUSY spectra. The resulting structure has significant differences from previous fits, and we hope will provide the "data" for developing models of fermion masses such as those based on a broken family symmetry.

M.S. acknowledges support from the United States Air Force Institute of Technology. The views expressed in this letter are those of the authors and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the US Government.

References

[1] W.-M. Yao, *Review of Particle Physics*, *Journal of Physics G* **33** (2006) 1+.

[2] B. V. Martemyanov and V. S. Sopov, *Light quark mass ratio from dalitz plot of $\eta \to \pi^+\pi^-\pi^0$ decay*, *Phys. Rev.* **D71** (2005) 017501, [hep-ph/0502023].

[3] T. E. W. Group, *A combination of cdf and d0 results on the mass of the top quark*, hep-ex/0703034.

[4] R. G. Roberts, A. Romanino, G. G. Ross, and L. Velasco-Sevilla, *Precision test of a fermion mass texture*, *Nucl. Phys.* **B615** (2001) 358–384, [hep-ph/0104088].

[5] D. Groom, *Review of Particle Physics*, *The European Physical Journal* **C15** (2000) 1+.

[6] H. Fusaoka and Y. Koide, *Updated estimate of running quark masses*, *Phys. Rev.* **D57** (1998) 3986–4001, [hep-ph/9712201].

[7] H. Baer, J. Ferrandis, K. Melnikov, and X. Tata, *Relating bottom quark mass in dr-bar and ms-bar regularization schemes*, *Phys. Rev.* **D66** (2002) 074007, [hep-ph/0207126].

[8] V. Lubicz, *Lattice qcd, flavor physics and the unitarity triangle analysis*, hep-ph/0702204.

[9] P. H. Chankowski and S. Pokorski, *Quantum corrections to neutrino masses and mixing angles*, *Int. J. Mod. Phys.* **A17** (2002) 575–614, [hep-ph/0110249].

[10] P. Ramond, *Journeys beyond the standard model*, . Reading, Mass., Perseus Books, 1999.

[11] V. D. Barger, M. S. Berger, and P. Ohmann, *Supersymmetric grand unified theories: Two loop evolution of gauge and yukawa couplings*, *Phys. Rev.* **D47** (1993) 1093–1113, [hep-ph/9209232].

[12] A. J. Buras, J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, *Aspects of the grand unification of strong, weak and electromagnetic interactions*, *Nucl. Phys.* **B135** (1978) 66–92.

[13] S. M. Barr and I. Dorsner, *Atmospheric neutrino mixing and b - tau unification*, *Phys. Lett.* **B556** (2003) 185–191, [hep-ph/0211346].

[14] H. Georgi and C. Jarlskog, *A new lepton - quark mass relation in a unified theory*, *Phys. Lett.* **B86** (1979) 297–300.

[15] J. L. Diaz-Cruz, H. Murayama, and A. Pierce, *Can supersymmetric loops correct the fermion mass relations in su(5)?*, *Phys. Rev.* **D65** (2002) 075011, [hep-ph/0012275].

[16] D. M. Pierce, J. A. Bagger, K. T. Matchev, and R.-j. Zhang, *Precision corrections in the minimal supersymmetric standard model*, *Nucl. Phys.* **B491** (1997) 3–67, [hep-ph/9606211].

[17] T. Blazek, S. Raby, and S. Pokorski, *Finite supersymmetric threshold corrections to ckm matrix elements in the large tan beta regime*, *Phys. Rev.* **D52** (1995) 4151–4158, [hep-ph/9504364].

[18] M. Carena, D. Garcia, U. Nierste, and C. E. M. Wagner, *Effective lagrangian for the anti-t b h+ interaction in the mssm and charged higgs phenomenology*, *Nucl. Phys.* **B577** (2000) 88–120, [hep-ph/9912516].

[19] M. Carena and H. E. Haber, *Higgs boson theory and phenomenology. ((v))*, *Prog. Part. Nucl. Phys.* **50** (2003) 63–152, [hep-ph/0208209].

[20] K. Tobe and J. D. Wells, *Revisiting top-bottom-tau yukawa unification in supersymmetric grand unified theories*, *Nucl. Phys.* **B663** (2003) 123–140, [hep-ph/0301015].

[21] K. Hagiwara, A. D. Martin, D. Nomura, and T. Teubner, *Improved predictions for g-2 of the muon and alpha(qed)(m(z)**2)*, hep-ph/0611102.

[22] L. L. Everett, G. L. Kane, S. Rigolin, and L.-T. Wang, *Implications of muon g-2 for supersymmetry and for discovering superpartners directly*, *Phys. Rev. Lett.* **86** (2001) 3484–3487, [hep-ph/0102145].

[23] D. Stockinger, *The muon magnetic moment and supersymmetry*, hep-ph/0609168.

[24] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B557** (1999) 79–118, [hep-th/9810155].

[25] L. J. Hall, R. Rattazzi, and U. Sarid, *The top quark mass in supersymmetric so(10) unification*, *Phys. Rev.* **D50** (1994) 7048–7065, [hep-ph/9306309].

[26] S. Komine and M. Yamaguchi, *Bottom-tau unification in susy su(5) gut and constraints from b –¿ s gamma and muon g-2*, *Phys. Rev.* **D65** (2002) 075013, [hep-ph/0110032].

[27] C. Pallis, *b - tau unification and sfermion mass non-universality*, *Nucl. Phys.* **B678** (2004) 398–426, [hep-ph/0304047].

[28] M. R. Ramage and G. G. Ross, *Soft susy breaking and family symmetry*, *JHEP* **08** (2005) 031, [hep-ph/0307389].

[29] S. F. King and M. Oliveira, *Yukawa unification as a window into the soft supersymmetry breaking lagrangian*, *Phys. Rev.* **D63** (2001) 015010, [hep-ph/0008183].

[30] T. Blazek, R. Dermisek, and S. Raby, *Predictions for higgs and susy spectra from so(10) yukawa unification with mu ¿ 0*, *Phys. Rev. Lett.* **88** (2002) 111804, [hep-ph/0107097].

[31] T. Blazek, R. Dermisek, and S. Raby, *Yukawa unification in so(10)*, *Phys. Rev.* **D65** (2002) 115004, [hep-ph/0201081].

[32] D. Auto, H. Baer, C. Balazs, A. Belyaev, J. Ferrandis, and X. Tata, *Yukawa coupling unification in supersymmetric models*, *JHEP* **06** (2003) 023, [hep-ph/0302155].

[33] C. Balazs and R. Dermisek, *Yukawa coupling unification and non-universal gaugino mediation of supersymmetry breaking*, *JHEP* **06** (2003) 024, [hep-ph/0303161].

[34] R. Gatto, G. Sartori, and M. Tonin, *Weak selfmasses, cabibbo angle, and broken su(2) x su(2)*, *Phys. Lett.* **B28** (1968) 128–130.

[35] K. Matsuda and H. Nishiura, *Can four-zero-texture mass matrix model reproduce the quark and lepton mixing angles and cp violating phases?*, *Phys. Rev.* **D74** (2006) 033014, [hep-ph/0606142].

[36] B. C. Allanach *et. al.*, *The snowmass points and slopes: Benchmarks for susy searches*, hep-ph/0202233.

[37] C. T. Hill, *Are There Significant Gravitational Corrections to the Unification Scale?*, *Phys. Lett.* **B135** (1984) 47.

[38] J. R. Ellis and M. K. Gaillard, *Fermion Masses and Higgs Representations in SU(5)*, *Phys. Lett.* **B88** (1979) 315.